

EQUILIBRIUM SHAPE CONTROL IN CIT PF DESIGN*

D. J. Strickler

Fusion Engineering Design Center, Oak Ridge National Laboratory
Oak Ridge, Tennessee 37831-8218

N. Pomphrey and S. C. Jardin

Princeton Plasma Physics Laboratory, P.O. Box 451, Princeton, New Jersey 08544

ABSTRACT

The free-boundary equilibrium code VEQ provides equilibrium data that are used by the Tokamak Simulation Code (TSC) in design and analysis of the poloidal field (PF) system for the Compact Ignition Tokamak (CIT). VEQ serves as an important design tool for locating the PF coils and defining coil current trajectories and control systems for TSC. In this paper, VEQ and its role in the TSC analysis of the CIT PF system are described.

INTRODUCTION

The PF system for the CIT is determined through a design and analysis process using the free-boundary equilibrium code VEQ¹ and the TSC.² TSC is a plasma transport and electromagnetics code that tests the consistency of the PF system with design physics and engineering guidelines. It uses preprogrammed PF coil currents and control matrices to characterize feedback systems for controlling plasma position and shape. VEQ provides these equilibrium data for TSC fiducial discharge calculations³ and plasma disruption modeling.⁴ The first step in the design process is to generate a detailed table of plasma geometry (e.g., elongation κ and triangularity δ), profile parameters (e.g., plasma internal inductance $l_i/2$ and poloidal beta β_p), and volt-second requirements at N representative points in time where preprogrammed currents are needed for a simulation. This table is often based on a preliminary TSC analysis. The next step is to calculate N reference equilibria and the associated coil current distributions, constraining the parameters in the table. Here VEQ computes PF coil current distributions of minimum stored energy while satisfying constraints on, for example, plasma shape, profiles, coil current magnitude, and volt-seconds. The PF currents are then varied about their reference values in a set of free-boundary, unconstrained equilibria to compute plasma shape control matrices relating shape perturbations to correction currents, $d\mathbf{I} = \mathbf{A}\epsilon$, where the elements of ϵ are typically errors in major radius R_0 , minor radius a , κ and δ at the 95% flux surface (κ_{95}

and δ_{95}), and flux linkage. This analysis is repeated until a solution is found that is consistent with plasma ignition criteria, physics guidelines on plasma shape and profiles, divertor sweep scenario, and volt-seconds for an adequate burn time.

The Equilibrium Package VEQ

In VEQ, a free-boundary equilibrium code is called from the numerical software packages HYBRD1⁵ and VMCON⁶ to compute values of free parameters (e.g., PF coil currents) in the equilibrium problem to constrain certain plasma shape and profile parameters (e.g., plasma radii, X-point position, or divertor strike points). Some of these shape constraints are shown in Fig. 1. When the number of free parameters is equal to the number of constraints, the package HYBRD1 is used to solve the nonlinear equations problem NLEQ:

$$\mathbf{F}(\mathbf{x}) = 0,$$

where \mathbf{F} and \mathbf{x} are vectors of length N ($\mathbf{F}:R^N \rightarrow R^N$). Whenever the number of independent variables

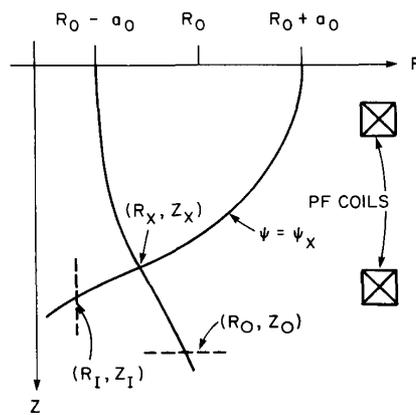


Fig. 1. VEQ equilibrium shape constraints include major radius, minor radius, X-point coordinates, and divertor strike points (I and O).

*Research sponsored by the Office of Fusion Energy, U.S. Department of Energy, under contract DE-AC05-84OR21400 with Martin Marietta Energy Systems, Inc.

is greater than the number of constraints (e.g., when the problem is to find seven CIT PF coil currents to fix R_0 , a , X-point coordinates, and flux linkage), it is possible to minimize some objective function (e.g., the PF coil energy $W_{PF} = 1/2\mathbf{I}^T\mathbf{M}\mathbf{I}$) while still satisfying the constraints. For this problem, the optimization package VMCON is used. VMCON is designed to solve the general nonlinear optimization problem NLOP:

$$\text{Minimize } f(\mathbf{x}),$$

subject to

$$F_i(\mathbf{x}) = 0, i = 1, \dots, k,$$

and

$$F_i(\mathbf{x}) \geq 0, i = k + 1, \dots, M,$$

where $f:R^N \rightarrow R^1$ and $F_i:R^N \rightarrow R^1$ are assumed to be continuously differentiable. In addition, VMCON allows simple bounds to be placed on independent variables. Evaluating the functions \mathbf{F} , f , and F_i requires solving the equilibrium problem. Here we refer to the equilibrium problem as EQP and to the HYBRD1 and VMCON problems as NLEQ and NLOP, respectively.

Coil Current Calculation

An axisymmetric PF coil with a rectangular cross section is represented in VEQ as a set of filamentary loops. VEQ has three major options for specifying or calculating coil currents. Subsets of coil currents may be

- Designated to have fixed current (option O1).
- Related linearly to the external poloidal flux at points on a prescribed plasma boundary contour (option O2). These currents are computed by solving a least-squares approximation problem with a regularization term. This calculation is part of the inner loop in the solution of the EQP.
- Related through a set of nonlinear equations to constraints on plasma X-point coordinates, divertor strike points, PF volt-seconds, or coil energy (option O3). With this option, the currents are treated as fixed in the solution of the EQP and as independent variables in the solution of either the NLEQ or the NLOP. The equilibrium subprogram is called iteratively by the appropriate software package to solve the nonlinear equation/minimization problem for the designated coil currents.

A given VEQ calculation, therefore, may have coil currents that are fixed, computed within the equilibrium loops (EQP), or determined by a software routine calling the equilibrium subprogram (HYBRD1 or VMCON).

Option O3 may be used, for example, to constrain the separatrix flux surface of a symmetric divertor plasma boundary to pass through two points on the midplane $(R_0 - a, 0)$ and $(R_0 + a, 0)$, and to intersect prescribed inner and outer divertor strike points (R_I, Z_I) and (R_O, Z_O) , as shown in Fig. 1. In addition, we may require a given PF flux linkage Φ_0 . When this option is used, one pair of outboard vertical field coil currents is usually adjusted within the EQP using option O2 to make the separatrix flux surface pass through $(R_0 + a, 0)$. If the number of remaining free currents is equal to the number of constraints (four in this case), they are determined by HYBRD1 as roots of the equation

$$\mathbf{F}(\mathbf{x}) = 0,$$

where $\mathbf{x}^T = (I_1, I_2, I_3, I_4)$ and

$$\mathbf{F} = \begin{bmatrix} (\psi_0 - \psi_x)/(\psi_L - \psi_0) \\ (\psi_I - \psi_x)/(\psi_L - \psi_0) \\ (a - a_0)/a_0 \\ (\Phi - \Phi_0)/\Phi_0 \end{bmatrix}.$$

Here a_0 is a given minor radius; ψ_I and ψ_0 are the values of the poloidal flux at the inboard and outboard strike points, respectively; ψ_L and ψ_0 are the flux at the plasma edge and magnetic axis, respectively; and $\Phi = \sum M_{i,p} I_i$ is the PF flux linkage with the plasma.

If the number of remaining free coil currents designated by O3 is greater than the number of constraints, VEQ switches to VMCON, which minimizes an objective function such as the stored energy,

$$f(\mathbf{x}) = 1/2\mathbf{x}^T\mathbf{M}\mathbf{x},$$

where $\mathbf{x}^T = (I_1, \dots, I_7)$, subject to the constraints $\mathbf{F}(\mathbf{x}) = 0$.

Profile Functions

Typical plasma pressure and toroidal magnetic flux profile functions used in HEQ in the evaluation of the plasma current distribution,

$$J_\phi = R dP/d\psi + F(dF/d\psi)/(\mu R), \quad (4)$$

are given in terms of their derivatives:

$$dP/dx = P_0[\exp(-Ax) - \exp(-A)]/[\exp(-A) - 1], \quad (5)$$

$$dF^2/dx = 2\mu R_0^2 P_0(1/\beta_J - 1)[\exp(-Bx) - \exp(-B)]/[\exp(-B) - 1], \quad (6)$$

where $x = (\psi - \psi_0)/(\psi_L - \psi_0)$. The parameter P_0 is scaled in the outer loop of the equilibrium algorithm so that the total plasma current,

$$I_p = \int \int J_\phi(x) d\Omega,$$

is fixed at an input value. In general, β_J controls the plasma beta and determines whether the plasma is paramagnetic ($\beta_J < 1$) or diamagnetic ($\beta_J > 1$), and A and B control the width of the current profile. Plasma pressure and current density profiles are shown as a function of radius R ($Z = 0$) in Fig. 2(a) for a $\langle\beta\rangle = 3\%$ CIT divertor equilibrium with $A = B = -1.04$ and $\beta_J = 0.59$ and in Fig. 2(b) for a $\langle\beta\rangle = 5\%$ case with $A = B = -1.0$ and $\beta_J = 1.05$.

Plasma equilibrium pressure and current density profiles are characterized by the poloidal beta,

$$\beta_p = 4 \int P dV / (\mu R_0 I_p^2),$$

and the plasma internal inductance,

$$l_i/2 = \int B_p^2 dV / (\mu^2 R_0 I_p^2),$$

respectively. A nominal value for the CIT poloidal beta at end of burn (EOB) is $\beta_p = 0.5$, and current profiles are referred to as broad for $l_i/2 = 0.3$, normal for $l_i/2 = 0.4$, and peaked for $l_i/2 = 0.5$.

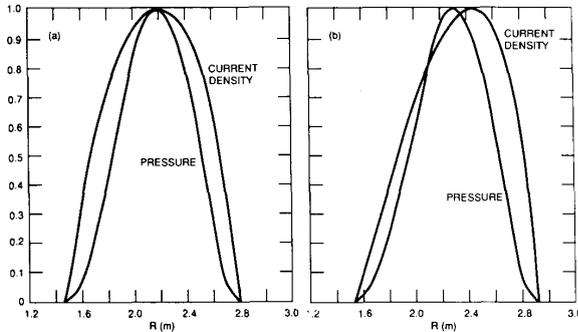


Fig. 2. Plasma pressure and normal current density profiles ($l_i/2 = 0.36$) for CIT divertor equilibria with (a) $\beta_p = 0.48$ and (b) $\beta_p = 0.89$.

The CIT PF System

The geometry considered in the following examples is based on a CIT design with $R_0 = 2.138$ m, $a = 0.662$ m, field on axis $B_t = 11$ T, and plasma current $I_p = 12.3$ MA. The external PF coil system used is GEM-28⁷ (Table I) and consists of seven coil groups, labeled PF1-PF7, that provide the equilibrium vertical field, shaping field, and inductive flux for an elongated

Table I. Poloidal field coil set (upper half plane) based on GEM-28

Coil	R_c (m)	Z_c (m)	ΔR (m)	ΔZ (m)
PF1	0.5917	0.4104	0.4064	0.8208
PF2	0.5917	1.3070	0.4064	0.7700
PF3	0.5917	2.1790	0.4064	0.7700
PF4	1.6300	2.8910	0.5629	0.4232
PF5	3.1900	2.8700	0.2913	0.2710
PF6	4.1870	2.0000	0.2753	0.3244
PF7	4.2245	0.7494	0.3543	0.4477

divertor plasma. The CIT PF system includes windings internal to the toroidal field (TF) coils; these windings are reserved for dynamic control of shape variations caused by rapid changes in plasma pressure and profiles and carry much smaller currents than the external coils.

The central solenoid stack is split into three sections (PF1, PF2, and PF3) for added flexibility in providing for a field null at startup and shaping the plasma cross section through a discharge. The outer ring coils, PF6 and PF7, provide the major component of the equilibrium vertical field, and PF6 also contributes to the shaping field or higher-order derivatives of the external field. In general, all external PF coils contribute to the equilibrium, shaping, and control of the CIT plasma, as well as to the flux change that induces the plasma current and ohmically heats the plasma.

Divertor Solution

A high-beta ($\langle\beta\rangle = 5\%$), diverted solution provides a "highly severe," but credible, case as a starting point for a TSC disruption analysis (Ref. 4). This section describes such a fixed X-point divertor solution with prescribed volt-seconds and β_p . Further equilibrium constraints (Table II) include plasma position, current, and field on axis.

For this problem, VEQ is used in an X-point-limited mode, where the plasma-vacuum boundary is defined by the flux value at a magnetic separatrix. The plasma

Table II. Fixed plasma parameters for VEQ divertor solution

Major radius (m)	2.138
Minor radius (m)	0.662
R , X-point (m)	1.660
Z , X-point (m)	1.480
Plasma current (MA)	-12.300
Field on axis (T)	11.000
Poloidal beta	0.890
Flux linkage (V·s)	38.000

boundary is fixed at only one point $(R_0 + a, 0)$ in the EQP, and HYBRD1 is used to compute four coil currents and one profile parameter (β_J) to constrain X-point coordinates, volt-seconds, a , and β_p . That is, the NLEQ problem is given by

$$F(\mathbf{x}) = 0,$$

and $\mathbf{x}^T = (I_{PF1}, I_{PF2}, I_{PF3}, I_{PF4}, \beta_J)$,

$$F = \begin{bmatrix} (a - a_0)/a_0 \\ (R_X - R_{X,0})/R_{X,0} \\ (Z_X - Z_{X,0})/Z_{X,0} \\ (\Phi - \Phi_0)/\Phi_0 \\ (\beta_p - \beta_{p,0})/\beta_{p,0} \end{bmatrix},$$

where

- I_{PF1}, \dots, I_{PF4} are independent variables in the NLEQ problem (option O3),
- $I_{PF5} = I_{PF6} = 1$ MA are fixed currents in the EQP (option O1),
- I_{PF7} is a variable in the EQP (option O2).

The divertor solution is shown in Fig. 3. In this mode, care must be taken to ensure that the initial guess for the coil current vector produces a solution with an X-point within the computational mesh Ω . This initial

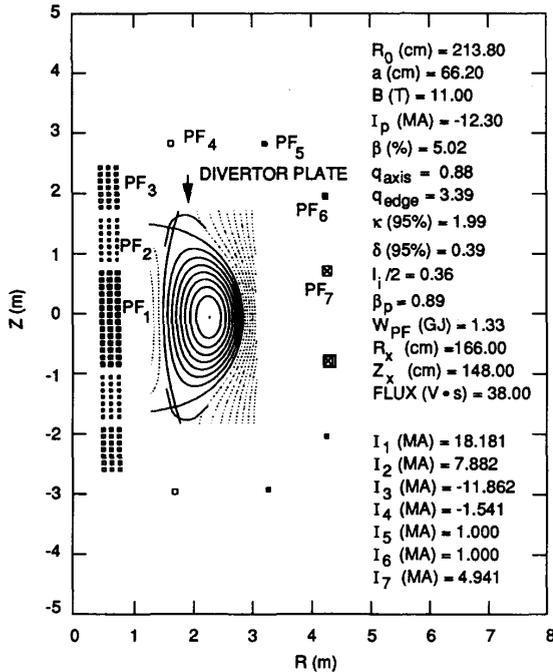


Fig. 3. VEQ divertor equilibrium solution.

guess is usually estimated from a previous solution or, in the case of a new problem, from a preliminary limiter solution.

Minimum-Energy Solutions

A critical point in the CIT coil current trajectories is at start of flattop (SOFT), where the 12.3-MA plasma is fully elongated and at relatively low triangularity (starting the divertor sweep). At this point in the scenario, large currents of opposite direction in solenoid elements PF2 and PF3 produce high separating forces on the solenoid sections. VEQ is exercised in the NLOP mode to examine the dependence of the coil current distribution and stored energy on the magnitude of I_{PF3} in a sequence of constrained minimum-energy equilibria. Constrained parameters and values for the equilibria of this section are listed in Table III.

Table III. Fixed parameters for VEQ minimum-energy solutions

Major radius (m)	2.138
Minor radius (m)	0.662
Elongation (95%)	2.000
Triangularity (95%)	0.300
Plasma current (MA)	-12.300
Field on axis (T)	11.000
Flux linkage (V*s)	36.800

As in the previous section, solutions are assumed to be X-point limited, and the plasma boundary is fixed at $(R_0 + a, 0)$ in the EQP by varying I_{PF7} . For the solutions in this section, however, six coil currents $(I_{PF1}, \dots, I_{PF6})$ are allowed to vary to minimize coil stored energy while constraining only four parameters— a , κ_{95} , δ_{95} , and flux linkage Φ . That is, the NLOP is formulated as

$$\text{Minimize } f(\mathbf{x}) = \mathbf{x}^T M \mathbf{x}, \mathbf{x}^T = (I_{PF1}, \dots, I_{PF6}),$$

subject to the constraints $F_i = 0$, where

$$\begin{aligned} F_1 &= (a - a_0)/a_0, \\ F_2 &= (\kappa_{95} - \kappa_{95,0})/\kappa_{95,0}, \\ F_3 &= (\delta_{95} - \delta_{95,0})/\delta_{95,0}, \\ F_4 &= (\Phi - \Phi_0)/\Phi_0, \end{aligned}$$

and M is the matrix of mutual inductances between the coils and plasma. In addition, the current in PF3 is bounded in magnitude.

Table IV gives the coil current distributions associated with a sequence of minimum-energy solutions at SOFT. For totally unbounded coil currents, as in case I of Table IV, currents in PF2 and PF3 are 21.9 MA and -16.7 MA, respectively, and magnetic stored energy is

Table IV. Minimum energy solutions at SOFT with current in coil PF3 unbounded, bounded by $|I_{PF3}| \leq 6$ MA, and bounded by $|I_{PF3}| \leq 2$ MA

	Coil current (MA)							Coil energy (MJ)
	PF1	PF2	PF3	PF4	PF5	PF6	PF7	
Case I, current in coil PF3 unbounded	15.62	21.87	-16.72	-7.75	3.05	3.90	2.20	1.94
Case II, current in coil PF3 bounded by $ I_{PF3} \leq 6$ MA	16.32	18.06	-6.00	-10.57	3.18	4.82	1.74	2.05
Case III, current in coil PF3 bounded by $ I_{PF3} \leq 2$ MA	16.62	16.53	-2.00	-11.46	2.89	5.51	1.46	2.17

$W_{PF} = 1.94$ GJ. With PF3 bounded in magnitude by $|I_{PF3}| \leq 2$ MA, the current in PF2 is 16.5 MA and $W_{PF} = 2.17$ GJ, because of larger shaping currents in the outboard coils. Therefore, a much more desirable distribution of current in the central solenoid is found by limiting the current in PF3, with only a small increase in stored energy.

Reference Equilibria for a Complete Discharge Simulation

In this section, VEQ is used to develop reference equilibria for the TSC 12.3-MA divertor fiducial discharge simulation. Preprogrammed PF coil current distributions are computed at 1-s intervals during current ramp (5.5–7.5 s) and through current flattop (7.5–12.5 s). The plasma is assumed to be diverted at 5.5 s, and the 95% elongation is held constant at 2.0 as the plasma current and toroidal field increase linearly to their maximum values of $I_p = -12.3$ MA and $B_t = 11$ T at 7.5 s. During the flattop interval, δ is increased in a controlled manner from 0.29 to 0.38 to produce the required divertor sweep. The outboard strike point sweeps 23 cm. Table V summarizes the values of the

parameters that are held constant with VEQ for the TSC fiducial equilibrium sequence. The equilibria at SOFT and EOB in Fig. 4 show the endpoints of the divertor sweep. The coil current trajectories are given in Fig. 5.

Plasma Shape Control Matrices

The TSC user often needs additional flexibility in specifying preprogrammed currents for a discharge simulation. Because the transport model in TSC evolves the plasma pressure and current density profiles self-consistently, these profiles are somewhat different from those assumed in the equilibrium calculations used to generate the coil currents, which results in differences in the plasma shape parameters. These differences are automatically corrected to some degree during a TSC calculation by feedback algorithms. It is extremely useful, however, to be able to make corrections between TSC runs without recalculating the equilibria. Here we introduce the concept of a shape control matrix A , relating correction currents in a subset of the PF coils to errors in the plasma shape:

$$d\mathbf{I} = \mathbf{A} \epsilon$$

Table V. Time points and plasma parameters held fixed for 12.3-MA divertor fiducial discharge reference equilibrium calculations ($R_0 = 2.138$ m, $a = 0.662$ m, $\kappa_{95} = 2.00$)

t (s)	5.5	6.5	7.5	8.5	9.5	10.5	11.5	12.5
δ_{95}	0.26	0.27	0.28	0.30	0.32	0.34	0.36	0.38
$l_i/2$	0.35	0.32	0.30	0.32	0.34	0.36	0.37	0.38
β_p	0.14	0.24	0.29	0.35	0.37	0.38	0.38	0.38
I_p (MA)	-9.05	-10.6	-12.3	-12.3	-12.3	-12.3	-12.3	-12.3
B_t (T)	9.4	10.2	11.0	11.0	11.0	11.0	11.0	11.0
Φ (V·s)	22.5	30.0	35.6	36.0	36.4	36.8	37.2	37.6

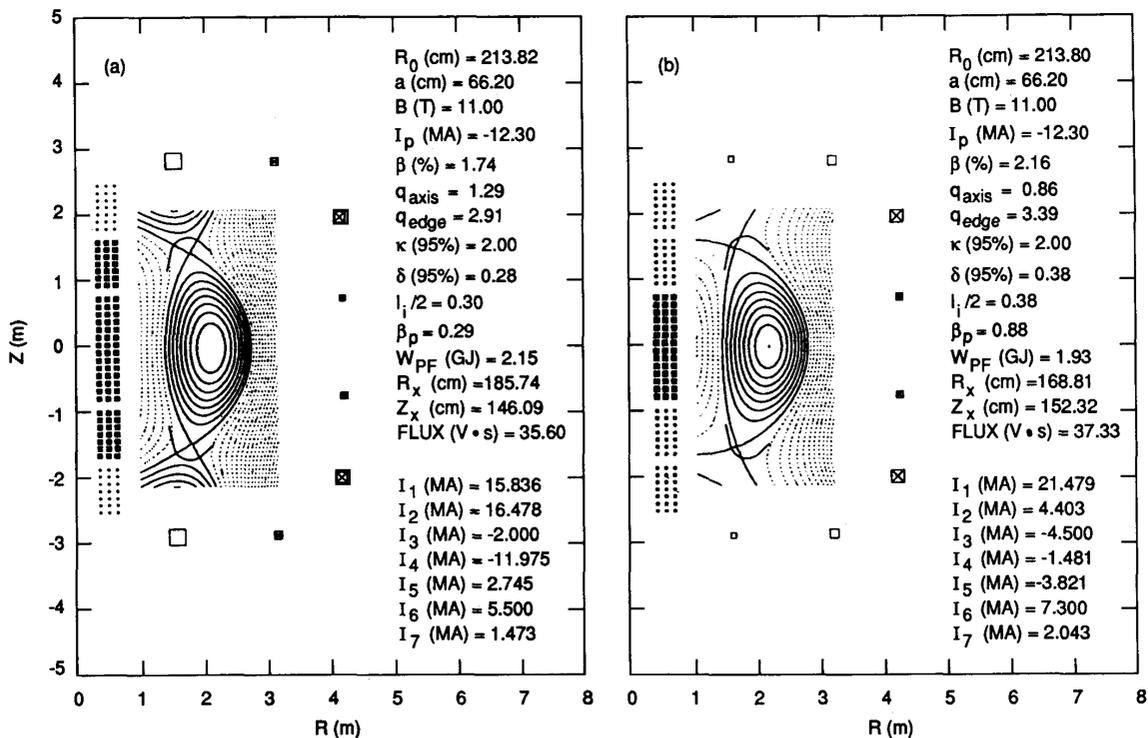


Fig. 4. Reference (a) SOFT and (b) EOB equilibria for a TSC discharge simulation.

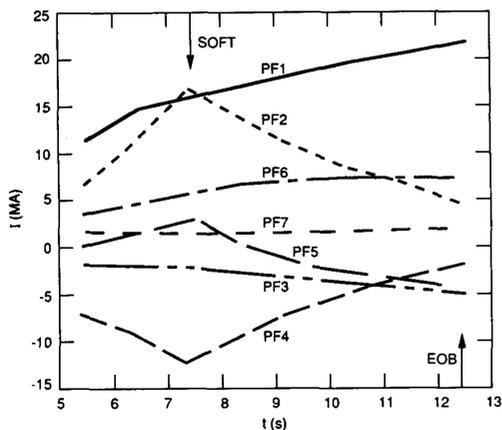


Fig. 5. Reference coil current waveforms for current flattop.

In CIT, the plasma shape control problem is assumed to consist of two subproblems:

1. how to detect errors in the plasma shape from flux loop measurements, and

2. how to determine the correction currents that best restore the plasma to its intended shape.

Solutions to the first subproblem, for a divertor plasma geometry, are being tested for speed (number of numerical operations) and accuracy; they will be reported later. The shape control matrix is proposed as a solution to the second subproblem and has proved useful in choosing preprogrammed coil current waveforms for the TSC fiducial discharge calculations. Here the elements of the error vector are chosen to be deviations from reference values in R_0 , a , κ_{95} , δ_{95} , and Φ . All seven of the external PF coils are used to control a smaller number of shape and flux parameters, so there may be an optimal solution to the second subproblem.

Shape control matrices associated with each reference equilibrium are developed by computing M additional free-boundary, unconstrained VEQ equilibria at each time point, changing the coil current distribution by $I_j = I_0 + dI_j$ between each calculation, where I_0 is the reference coil current distribution and the index j refers to the equilibrium calculation. For each calculation, the error vector ϵ_j , whose elements are the changes in certain shape parameters and flux linkage between equilibrium j and the reference equilibrium,

is recorded. If $d\mathbf{I}_j^T = [dI_{1j}, \dots, dI_{Nj}]$ represents the change in the current distribution associated with the j th equilibrium calculation, we set

$$\mathbf{a}_i \in_j \approx d\mathbf{I}_{ij}, i = 1, \dots, N, j = 1, \dots, M$$

and solve for the elements of the row vectors \mathbf{a}_i of the control matrix \mathbf{A} . The error vector usually contains five elements, resulting in an $N \times 5$ control matrix. Typically $M > 5$, and this system is solved as a least-squares problem.

In the X-point-limited calculations for the fiducial discharge simulation, $\epsilon^T = [\Delta R_0, \Delta a, \Delta \kappa_{95}, \Delta \delta_{95}, \Delta \Phi]$, and the coil currents are changed one at a time between equilibrium problems; that is, $\mathbf{I}_j^T = (I_{10}, \dots, I_{j0} + dI_j, \dots, I_{N0})$, where I_{i0} refers to the i th reference coil current in the subset of coils chosen to be used in the control matrix. A control matrix corresponding to case III in Table IV is given in Table VI. Each column of the control matrix corresponds to a current distribution that would change the plasma shape (or flux) parameter associated with that column while leaving the remaining parameters unchanged. To estimate the change in the current distribution of case III in Table IV for a 0.05 change in both elongation and triangularity, for example, one would multiply the sum of columns 3 and 4 by 0.05. The accuracy of the estimate decreases with the magnitude of the desired changes.

Summary

Axisymmetric magnetohydrodynamic equilibrium codes continue to play an important role in the design of tokamak experiments. This is particularly evident in the case of CIT, for which unique requirements in plasma shape control and divertor design have led to new developments in the computational equilibrium problem. The free-boundary equilibrium code VEQ,

designed to compute the PF coil current distribution of elongated, magnetically limited tokamak plasmas with minimum stored energy and specific constraints on plasma shape, profiles, and volt-seconds, is used extensively in the CIT PF coil design process to provide preprogrammed coil currents and plasma shape control matrices for the fiducial discharge calculations and plasma disruption modeling carried out with TSC.

References

1. D. J. Strickler, J. D. Galambos, and Y-K. M. Peng, *MHD Equilibrium Methods for ITER PF Coil Design and Systems Analysis*, ORNL/FEDC-88/7, Oak Ridge National Laboratory, March 1989.
2. S. C. Jardin, N. Pomphrey, and J. Delucia, "Dynamic Modeling of Transport and Positional Control of Tokamaks," *J. Comput. Phys.*, vol. 66, pp. 481-507, 1986.
3. N. Pomphrey, *The 11 MA Fiducial Discharge*, AE-890417-PPL-04 TR-007, Princeton Plasma Physics Laboratory, April 1989.
4. R. O. Sayer, Y-K. M. Peng, D. J. Strickler, and S. C. Jardin, *TSC Disruption Scenarios and CIT Vacuum Vessel Force Evolution*, ORNL/FEDC-89/3, Oak Ridge National Laboratory, to be published.
5. J. J. Moré, B. S. Garbow, and K. E. Hillstrom, *User Guide for MINPACK-1*, ANL-80-74, Argonne National Laboratory, 1980.
6. R. L. Crane, K. E. Hillstrom, and M. Minkoff, *Solution of the General Nonlinear Programming Problem with Subroutine VMCON*, ANL-80-64, Argonne National Laboratory, 1980.
7. R. D. Pillsbury, "A Preliminary PF Current Waveform for a 12.3 MA Diverted Discharge in the 2.138 m Bucked Design for CIT," MIT-PF050389 (GEM-28), Massachusetts Institute of Technology, May 3, 1989.

Table VI. Plasma shape control matrix for the solution case III in of Table IV

Coil	R_0 (MA/m)	a (MA/m)	κ_{95} (MA)	δ_{95} (MA)	Φ (MA/V·s)
PF1	17.22	-6.73	14.52	16.93	0.58
PF2	1.86	-46.65	-54.84	-43.61	0.73
PF3	-5.65	146.79	58.47	2.63	0.34
PF4	-8.29	64.47	27.23	9.85	0.22
PF5	-0.69	-11.54	-5.57	0.36	0.01
PF6	-0.22	-5.61	-2.27	-0.23	0.00
PF7	-0.29	-5.47	-1.11	-0.44	0.00